Using the "Moment of Inertia Method" to Determine Product of Inertia

by

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About the Authors

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1 ABSTRACT Product of inertia is generally measured using a spin balance machine. In this type of machine, the object is rotated at a speed of about 100 RPM, and the reaction forces against the upper and lower spindle bearings are measured. Product of inertia is then calculated automatically by the machine's on line computer, using formulas that involve the vertical spacing between the upper and lower bearings, and the height of the object above the mounting surface of the machine. Objects such as control fins and satellites with extended solar panels cannot be measured using this method, because of the large, non-repeatable errors which are introduced by the entrained and entrapped air and turbulence. This paper outlines a method of determining product of inertia by making a series of moment of inertia measurements with the object oriented in 6 different positions. Product of inertia can then be calculated using formulas which involve the rotation angles of the different fixture positions. Moment of inertia is measured by oscillating the object on a torsion pendulum. Since the object moves very slowly during this measurement, there are negligible centrifugal and windage forces exerted on the object. Furthermore, the mass of the entrapped and entrained air can be compensated for by making a second set of measurements in helium, and extrapolating the data to predict the mass properties in a vacuum.

This paper gives step-by-step instructions on how to measure product of inertia on a torsion pendulum. Special fixtures must be constructed to move the object to the six positions while keeping both the object and the fixture CG near the center of oscillation. We have included design details of such a fixture. Since vacuum data was required, measurements were made in a chamber which could be filled with helium. The design of this chamber is also explained in detail.

To illustrate this method, we have used as an example real measurements which were made of airfoil control fins manufactured by one of our customers. For this example, we determined all mass properties: weight, center of gravity along three axes, moment of inertia about three axes, and product of inertia in three planes, all referred to vacuum conditions.

2 INTRODUCTION The effect of entrapped and entrained air is usually negligible when measuring objects that have high density. These objects also frequently have sufficient structural integrity so that they do not deflect due to centrifugal force when measured on a spin balance machine. Low density objects, however, cannot be accurately measured on a spin balance machine unless the machine and object are enclosed in a large vacuum chamber. Such a chamber is very expensive and extensive alterations are required to the spin balance machine to permit it to operate in a vacuum. This paper outlines an alternate method of measuring product of inertia ("POI"), in which the object is oscillated on a gas bearing torsion pendulum rather than measured on a spin balance machine.

Not all flight hardware lends itself to this alternate POI measurement method. This method is particularly useful for objects that have a low density, large cross sectional area projected in at least one plane, and small volume or open (porous) construction. These characteristics have the effect of maximizing the effect of entrained air (air that is dragged along with the object when it moves, adding to its effective mass). Typical candidates for this method are airfoils, control surfaces, and satellites with solar panels (or other large protrusions).

Figure 1 shows the airfoil used in this project.

Those parts that have a large enclosed volume present an additional set of problems. They typically have a large MOI error due to entrained (external) air which can be corrected for by helium measurements, but they also have a large error due to entrapped air. This entrapped air effect may or may not have to be corrected for depending on whether it remains in the test article during flight or if it dissipates. If it remains with the test article (sealed in), it need not be corrected for since it is a true component of the test part mass. If it dissipates when the object encounters the vacuum of

Figure 1

space, then some method of correcting for the air mass is necessary in order to predict the mass properties in a vacuum. Again, the helium method provides a means of correcting for this mass. Replacing the air in the test part with helium may place constraints on the fixture design, and additional time may be required to attain atmospheric equilibrium between inside and outside of the test part.
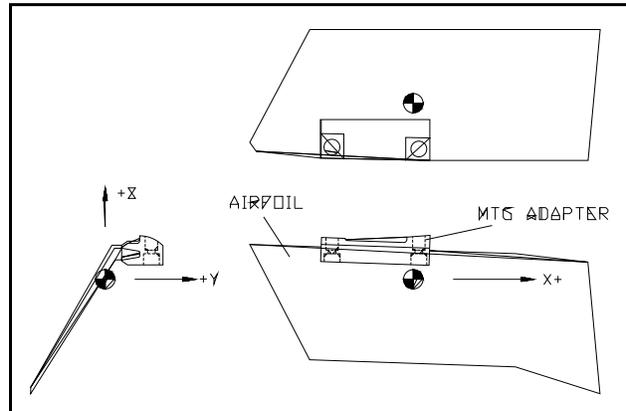
The helium method is described in the SAWE paper entitled "Using Helium to Predict the Mass Properties of an Object in the Vacuum of Space" by Richard Boynton, Robert Bell, and Kurt Wiener (paper number 2024). This paper was presented at the annual SAWE conference in San Diego, CA, 20-23 May, 1991.

The MOI method of POI determination requires the use of an extremely accurate moment of inertia instrument. This instrument must also be capable of measuring center of gravity. The only type of instrument which we know of that has the required accuracy is a gas bearing supported torsion pendulum such as the Space Electronics KSR Series instruments.

Typical POI accuracy required by airframe manufacturers is 1%. The typical accuracy obtained by the proposed MOI/Helium atmosphere method is on the order of 0.53%.

   TEST METHOD The basic procedure for measuring the POI of a two dimensional object is described in an earlier technical paper entitled "Determining Product of Inertia using a Torsion Pendulum" by Charles L. Jodry and Richard Boynton. (SAWE paper No. 1473, presented in San Jose, CA, 17-19 May, 1982). This paper expands this procedure for the three dimensional case, and adds considerable practical advice not found in the previous paper.

   NUMBER OF MOI MEASUREMENTS For the general case, the total number of MOI measurements needed for POI calculations is nine: three in each of three mutually perpendicular planes. If the intersections of these planes are selected to be the coordinate axes, then the MOI about each of these axes will be common to two planes, thus reducing the total number of measurements

to six: three about the X, Y, and Z axes, and three about axes at 45 degrees between the X-Y, Y-Z, and Z-X axes. If vacuum data is required, the same six MOI measurements must also be repeated in a helium atmosphere.

3 CALCULATIONS FOR 3 AXIS POI The coordinate system for the MOI method (fig. 2) has its origin (O) at the test part CG. The axes will be designated X, Y, and Z passing through the CG.

If the test part were fixtured so that it could be rotated through an angle C about a horizontal axis (i.e. the Z axis) and MOI measured about numerous axes in the X-Y plane, including the X and Y axes, the MOI would be found to vary sinusoidally. If the angle C ranges over 180 degrees, the maximum and minimum values of MOI can be seen in a plot of MOI vs C. (fig. 3).
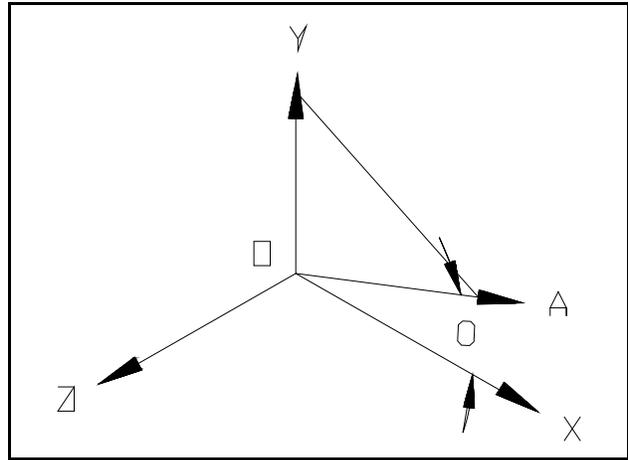
The axes about which the maximum and minimum MOIs are measured are the Principal Axes. For all other axes the moment of inertia

Figure 2

I_{Axy}, about an axis (A) in the X-Y plane at an angle C from the +X axis, and the product of inertia P_{xy}, are related through the equation:

$$I_{Axy} = I_{yy}\sin^2 C + I_{xx}\cos^2 C - P_{xy}\sin 2C \qquad\qquad \textbf{(1)}$$

Solving this equation for P_{xy} forms the basis for the MOI method of POI determination.

$$P_{xy} = \frac{(I_{yy}\sin^2 C + I_{xx}\cos^2 C - I_{Axy})}{\sin 2C} \qquad\qquad \textbf{(2)}$$
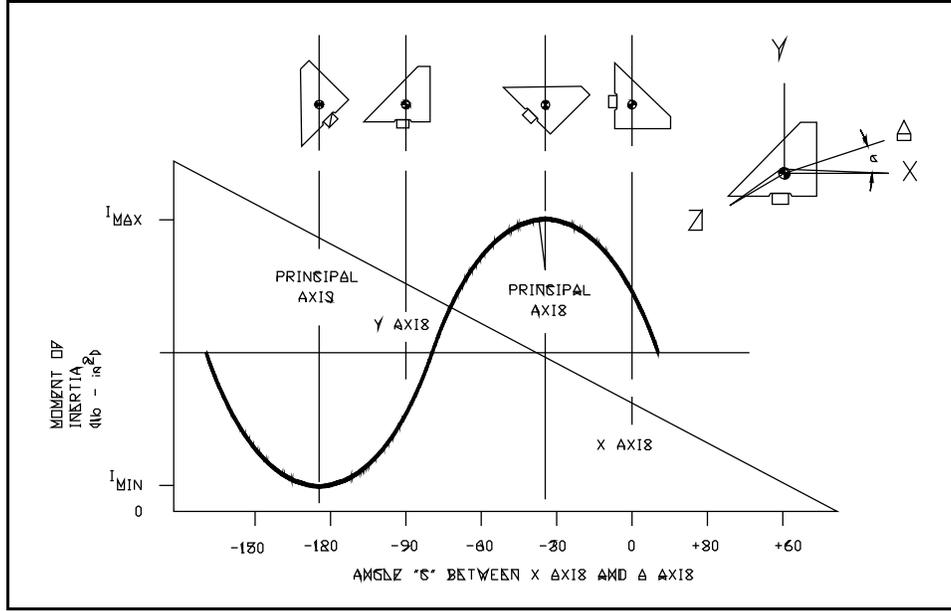
Figure 3

The POI in each of the three machine coordinate planes, X-Y, Y-Z, Z-X is calculated from MOI data taken about three axes in each plane. For best accuracy the three axes will be the two coordinate axes (i.e. X and Y in the X-Y plane) and an axis at 45 degrees between the coordinate axes. This third axis is referred to as the A axis and the MOI about this axis in the X-Y plane would be referred to as I_{Axy}. If, due to mechanical limitations, the 45 degree orientation cannot be achieved, any angle in the reference planes may be used. The accuracy of the calculated POI value will be degraded as the deviation from 45 degrees increases.

The equation used to calculate the POI in the X-Y plane when A is at 45 degrees is:

$$P_{xy} = \frac{I_{xx} + I_{yy}}{2} - I_{Axy} \qquad \textbf{(3)}$$

Similarly, the POI for the Y-Z and Z-X planes would be calculated from MOI data about axes in those planes such that :

$$P_{zx} = \frac{I_{zz} + I_{xx}}{2} - I_{Azx} \qquad \textbf{(4)}$$

and

$$P_{yz} = \frac{I_{yy} + I_{zz}}{2} - I_{Ayz} \qquad \textbf{(5)}$$

At this point, all MOI and POI values are known for the reference X, Y, Z coordinate system, therefore, they can be determined for any other coordinate system using the standard axis rotation

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and translation equations when the rotation angles are known.

The next step is to determine the orientation of the principal axes and the angle of inclination. This is not as straight forward as axis rotation and translation because the angles between the reference axes and the principal axes are not known.

Figure 4

PRINCIPAL AXIS ORIENTATION The most common need for principal axis information is to determine angle of inclination. For a two dimensional object in the X-Y plane (or an object where the X-Y plane is a plane of symmetry) the angle of inclination (P) between the X axis and the principal axis X', can be calculated from:
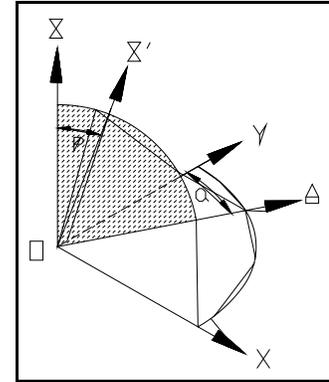
$$P = (.5)\arctan\left(\frac{2P_{xy}}{I_{yy} - I_{xx}}\right) \qquad (6)$$

For a three dimensional part, the same form of equation may be used, but the values of POI and MOI must be relative to axes which lie in a plane defined by the reference axis and corresponding inclined principal axis, i.e Z and Z'. Refer to figure 4 where the axes have been re-defined for the general case.

The Z-Z' plane will be at an angle (a) relative to the +X axis such that:
Let (O-A) be an axis in the X-Y plane at an angle (a) from the +X axis. It is, in fact, the intersection between plane Z-Z' and plane X-Y.

$$a = \arctan\left(\frac{P_{zy}}{P_{zx}}\right) \qquad (7)$$

The equation can now be re-written:

$$P = (.5)\arctan\left(\frac{2P_{ZA}}{I_A - I_Z}\right) \qquad (8)$$

From our original MOI measurements, only I_Z is known. However, we also know angle (a) so that we can use eq.1 to determine I_{AA} :

$$I_{AA} = I_{YY}\sin^2 a + I_{XX}\cos^2 a - P_{XY}\sin 2a \qquad (9)$$

and then determine

$$P_{ZA} = \sqrt{(P_{zx})^2 + (P_{zy})^2} \qquad (10)$$

The same method may be used to find the angle between the X and X' and Y and Y' axes. However,

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if the orientation of the principal axes and moments of inertia about the principal axes must all be known, it may be more practical to apply matrix solutions.

4 ACCURACY REQUIRED

Accuracy was required to be within the tolerances listed here. The measured values are within the tolerances shown in the "Actual" column. The uncertainties were determined from the error analysis presented later in this paper.

	REQUIRED	ACTUAL
WEIGHT	± 0.1%	± 0.01%
CG-X (in)	± 0.1 in	± 0.01 in
CG-Y (in)	± 0.1 in	± 0.01 in
CG-Z (in)	± 0.1 in	± 0.01 in
I_{XX}	± 1.0 %	± 0.35 %
I_{YY}	± 1.0 %	± 0.35 %
I_{ZZ}	± 1.0 %	± 0.35 %
P_{XY}	± 1.0 %	± 0.53 %
P_{YZ}	± 1.0 %	± 0.53 %
P_{ZX}	± 1.0 %	± 0.53 %

5 THE TEST PROCEDURE It is important to organize the measurement procedure to minimize:

● the total number of measurements to be made

● the number of tare measurements to be made

● the number of fixture setups to be used,

● the number of helium measurements to be made (if needed)

TARE MEASUREMENTS Since the orientation of the test part is different for each measurement, the locating fixture will also, in all probability, have a different MOI in each position. Therefore, a tare measurement for each position is also required. If the fixture is very repeatable, the tare measurements may only have to be made once and stored for use with successive part measurements. This may require a special software data storage and retrieval program since the typical standard program stores only one tare measurement.

FIXTURE SETUPS With six required measurements, six different fixtures could be used. It has been our experience that three fixtures are adequate. One fixture is designed to support the test part on a pivot along one axis (i.e. Y) so that three measurements can be made with another axis (i.e X) located at zero, 45, and 90 degrees from the vertical, resulting in three data points in the vertical Z-X plane. The second fixture supports the part on a pivot along the part Z axis and allows the part X axis to be at zero and 45 degrees to the vertical adding two data points in the X-Y plane. Finally, a third fixture locates the part X axis horizontal and the Y axis at 45 degrees to the

horizontal so MOI can be measured about an axis midway between the Z and Y axes giving the third data point in the Y-Z plane. Figures 5 & 5A show the 3 fixtures and the associated test positions used.

The three fixture assemblies are shown in figures 10, 11, and 12 later in the paper.

TEST POSITIONS The six MOI test positions are shown and identify the test sequence in Figure 5. CG and MOI were measured in each position so that MOI about CG could be determined.

The CG locations as measured in positions 1A, 3A, and 4B were recorded as the "official" test part CG location relative to the airfoil coordinate system datum.

Notice that even though there are 3 positions in each plane required for POI calculations, there are only six different positions since 3 positions are common to 2 planes. The duplicate positions are noted in figure 5. This is most apparent in the drawing for fixture C. Only the intermediate (45 degree) position need be measured since the Moment about Z was already measured in position 1 using fixture A, and MOI about axis Y was measured in position 4 using fixture B.

SEQUENCE OF MEASUREMENTS If the standard software is used or there is only one test part to measure, the sequence of tare and part measurements can be arranged to minimize the number of times the part must be mounted/dismounted as described below. The test positions are shown in figure 5.
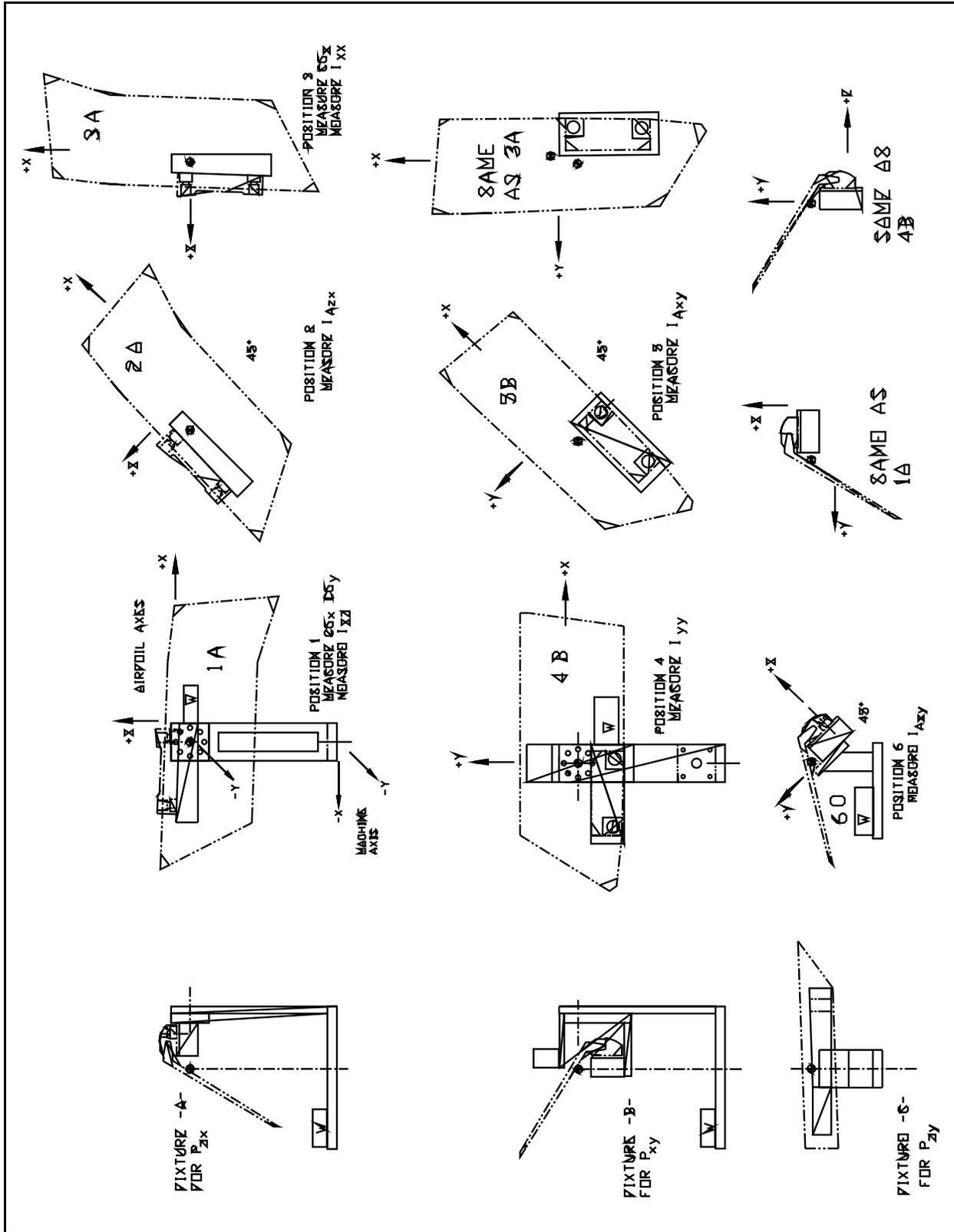
Figure 5
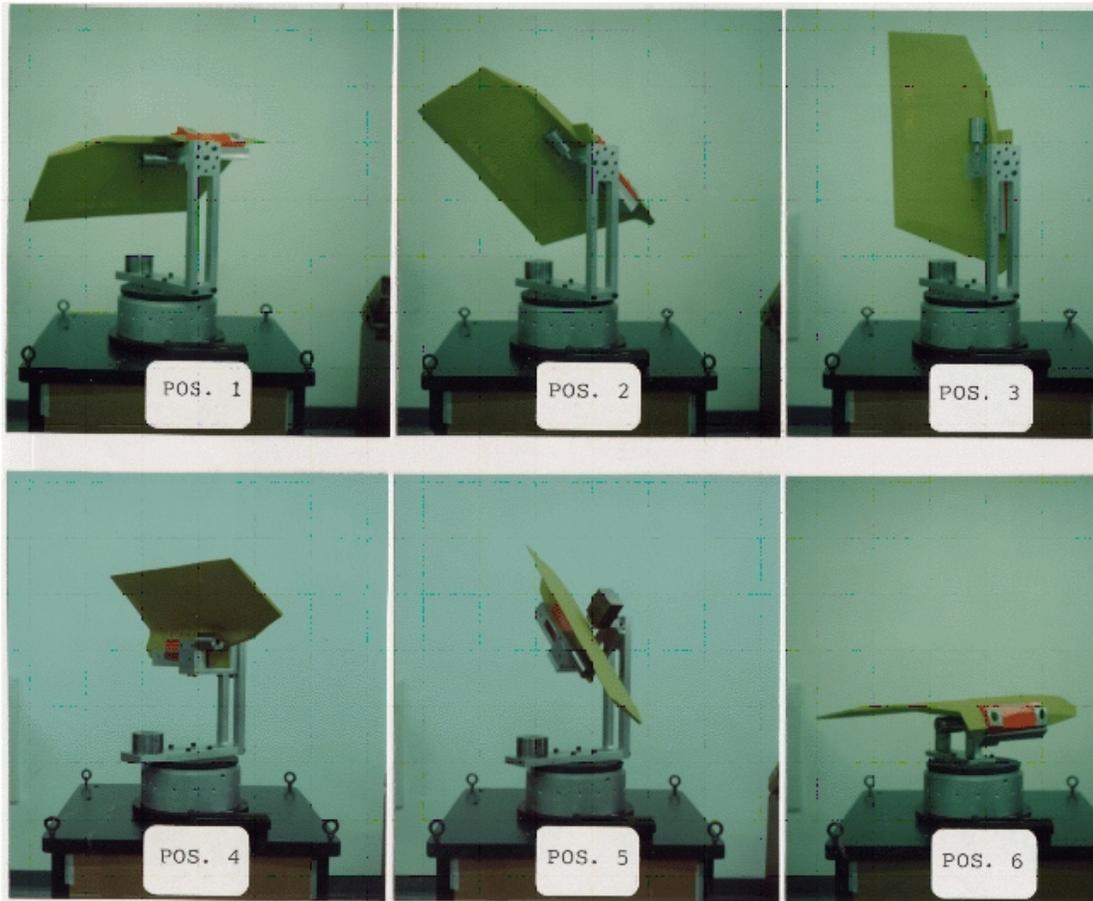
Figure 5A

1.   Setup fixture A, position 1, and measure tare position 1.
2.   Mount part, measure part position 1.
3.   Rotate part to position 2, measure part position 2.
4.   Remove part, measure tare position 2.
5.   Rotate fixture to position 3 measure tare, position 3.
6.   Mount part, measure part position 3.
7.   Setup fixture B, position 4, measure tare position 4.
8.   Mount part, measure part, position 4.
9.   Rotate to position 5, measure part position 5.
10.   Remove part, measure tare position 5.
11.   Setup fixture C, position 6, measure tare position 6.
12.   Mount part, measure part, position 6.

In each test position, the CG location as well as MOI should be measured. This will allow the machine to report MOI through the CG. CG location data is often required as well as MOI. If the nominal CG location is on an axis passing through the fixture pivot, and the CG location is very close to nominal, the error introduced in the MOI measurement will be small. This is because the MOI error is proportional to the ratio of the CG offset and radius of gyration squared. Typically, the radius of gyration for airfoils and similar shapes is large compared to the CG offset. If this ratio is smaller than 1/100 the MOI error due to CG offset will be negligible.

For example, let us assume the CG offset is 0.060 inches and the radius of gyration (K) is on the order of 6 inches, a ratio of 1/100. The ratio of CG offset to K squared is;

$$(0.06)^2/(6)^2 = 0.0036/36 = 0.0001$$
or
0.01%

This means that for most measurements of this type, the error due to CG offset for **any one** MOI measurement will be on the order of 0.01%. There is however an accumulation of error since three MOI measurements are used for each POI calculation. The design constraints on the fixture are also less severe if CG measurements are made for each MOI measurement.

6 HARDWARE The hardware required for these tests is:

● Mass Properties measuring system (i.e. Space Electronics Model KSR Instrument) including MOI/ CG instrument, computer, monitor, keyboard, & interfaces. An accurate weight platform is also required.

● Fixture system, including test part/fixture interface and fixtures to locate object in 6 positions

● Helium chamber system including helium supply, piping, flow control, and helium/oxygen monitor if vacuum data are required.

   INSTRUMENT USED This method requires a mass properties instrument which can measure both MOI and CG. We used the Space Electronics Model KSR-330. For the weight of test parts measured, (approximately 10 lb) this instrument is capable of measuring CG with an accuracy of 0.001 inch and measures moment of inertia with an accuracy of approximately 0.1 %.

W MARKS COUNTERWEIGHTS
USED TO BALANCE FIXTURES

FIXTURES -A- AND -B-
USE A COMMON FIXTURE BASE

FIXTURES -A- AND -B-
HAVE MATING PIVOT PLATES

FIXTURE -C- DOES NOT ROTATE

INTERFACE BLOCK

-Y

-X

W

FIXTURE -B-
FOR P_{xy}

INTERFACE BLOCK

-X

-Y

W

FIXTURE -C-
FOR P_{zy}

AIRFOIL AXES

+X

+Z

-Y

-X

MACHINE AXIS

PIVOT AXIS

INTERFACE BLOCK

FIXTURE BASE

W

FIXTURE -A-
FOR P_{zx}

Figure 7

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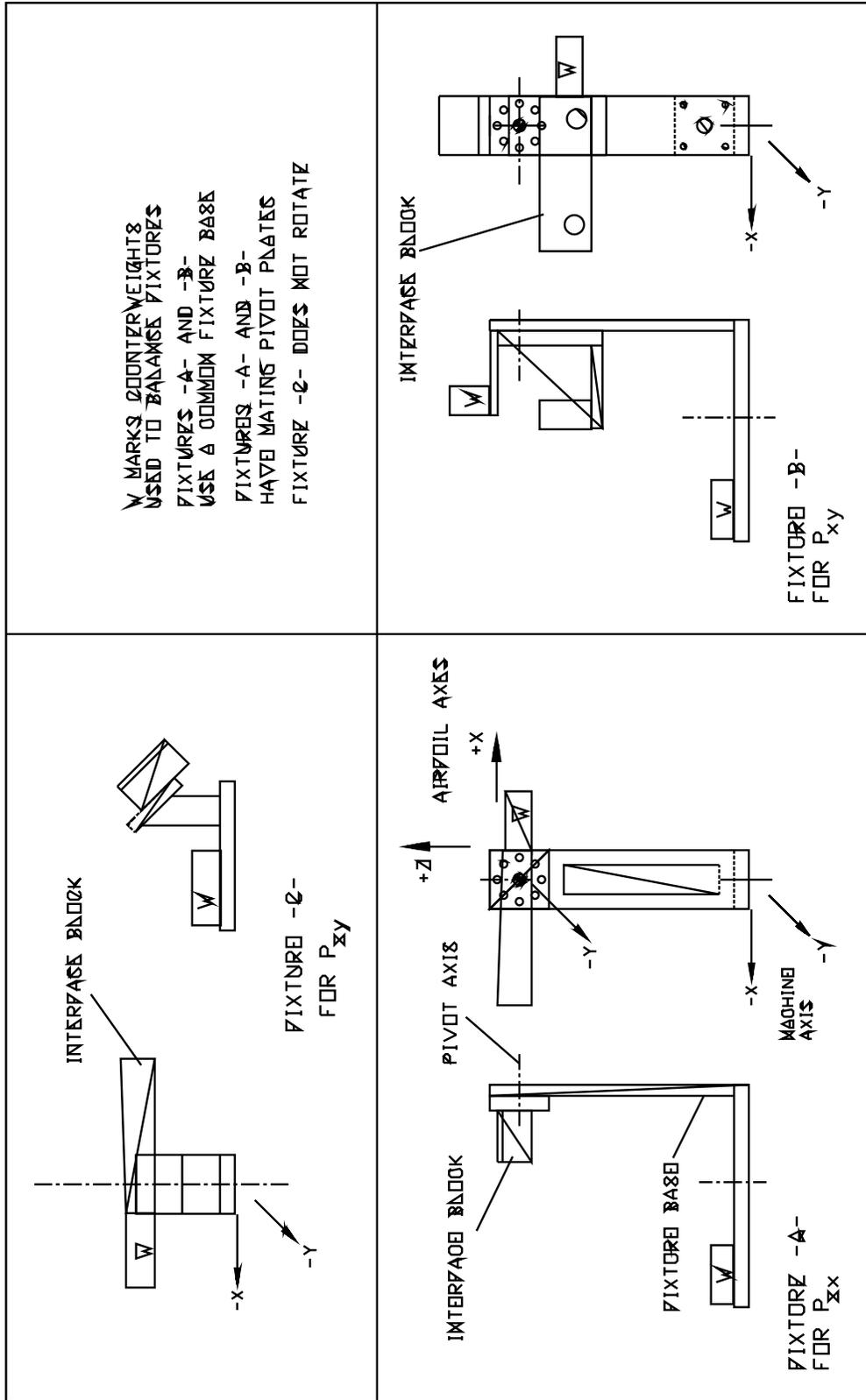
The instrument will report the MOI about the center of rotation of the test table and also about the CG of the test object.

Measurement accuracy is **always** degraded by fixture uncertainties and uncertainties in the test part/fixture interface.

FIXTURE SYSTEM The fixture system (see fig. 6) for a given test part configuration consisted of:
- one test part/fixture interface block
- one fixture base with pivot
- two pivoting locating fixtures
- one non pivoting fixture

Each of these elements is critical to the overall ease of operation and accuracy of the fixture system. Any non-repeatabilities or dimensional uncertainties in the critical dimensions will directly add to the CG locational uncertainty and indirectly to the MOI uncertainty.

It is the function of the test part/fixture interface block to mate with the test part and to establish three reference axes which are parallel to the test part datum axes. It must also mount to the fixture in such a way to orient the axes planes parallel to the machine reference axes at known offset distances.

The fixture base (with pivot) provides a sturdy support for the test part which mounts to the measuring instrument in a highly repeatable manner. If the test part geometry permits, this base may be used to support more than one of the locating fixtures. In the case under discussion it supported 2 locating fixtures.

The locating fixture elements serve the purpose of orienting the datum planes of the interface block correctly and mounting the test part on the base pivot. The locating fixture rotates with the part and interface block to each test position.

The non-rotating fixture serves the same purpose as the other locating fixture elements but does not rotate. It, therefore, orients the test part for only one MOI measurement. Specific design features are discussed later in this paper.

The fixtures were designed to place the nominal CG location of the airfoil directly over the center of oscillation. In addition, the fixtures were balanced so the CG of the bare fixture was also close to the center of oscillation. It was expected that the actual location of the CG would vary considerably from the nominal location so only the MOI about true CG was used for POI calculations.

7 FIXTURE DESIGN Fixture design for the MOI method of POI determination is typically constrained by six major considerations:

1.   The fixture should interface with the test part so the test part coordinate axes and the machine

coordinate axes are parallel.

2. The origin (datum) of the test part coordinate system must be positioned at a well defined, known location relative to the machine origin (center of machine rotation).

3. The fixture(s) must permit rotation or orientation of the test part to the six test positions in such a way to locate the nominal test part CG on the machine origin (center of machine rotation). That is, the test part must rotate about its nominal CG.

4. The fixture must provide adequate clearance for the test part above the test instrument as it rotates to each test position.

5. The fixture must have minimum weight and MOI consistent with adequate structural rigidity so that the difference in loading between tare and part measurement does not permit deflections which cause measurable errors in MOI or CG location.

6. The fixtures should be reasonably well balanced about the machine center in all test positions.

Airfoils, typically, do not have simple geometric shapes such as rectangles and cylinders. The coordinate axes of airfoils are well defined geometrically in terms of the air frame and theoretical aerodynamic contours. Unfortunately, loose manufacturing tolerances and complex geometry often make it very difficult to identify the datum accurately in terms of hardware details. Figure 7 shows the airfoil used in this project with its mounting adapter.
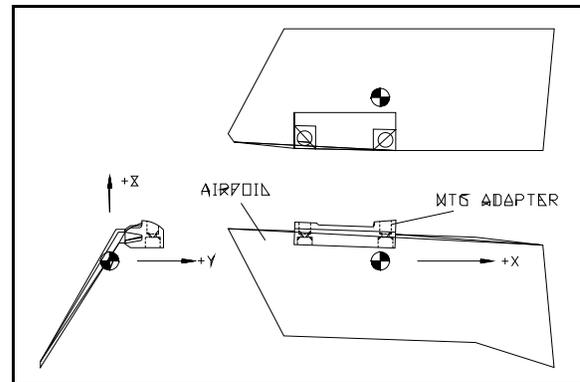
Figure 8

The mounting adapter is a piece of flight hardware which is fastened to the airfoil and is used to mount the airfoil to the airframe. It is considered an integral part of the airfoil for all tests.

The six measurement positions are shown with the associated fixtures in figure 5.
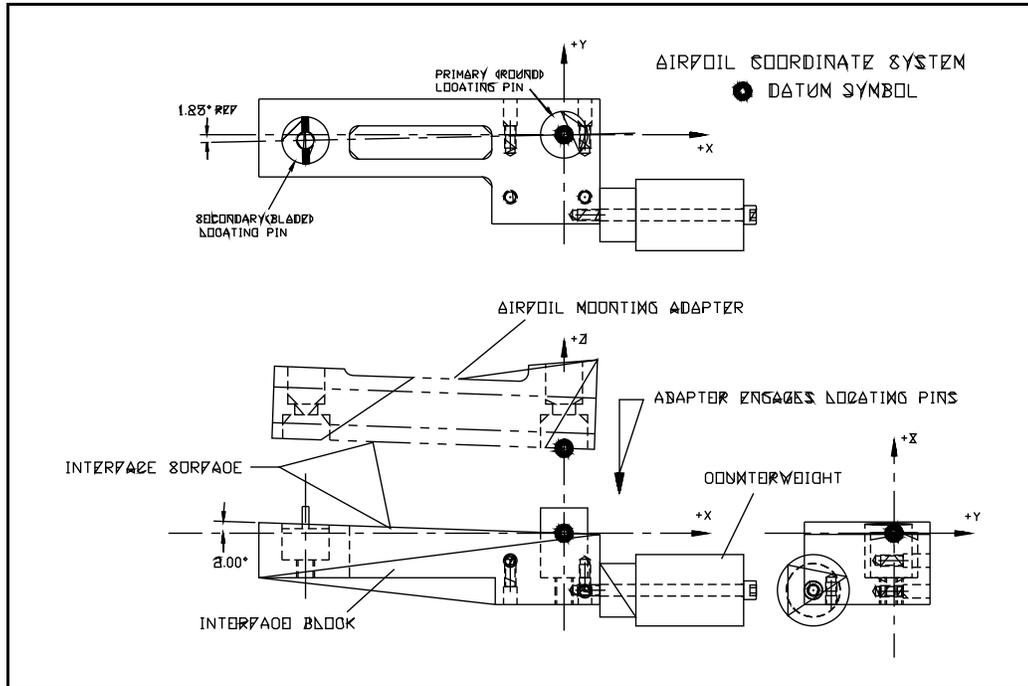
Figure 8

INTERFACE BLOCK An interface block (fig. 8) was devised to provide a rectangle with faces parallel to the airfoil coordinate system. It was relieved as much as possible to reduce weight without sacrificing structural rigidity. A counterweight was attached to the interface block so the fixture with the interface block was statically balanced about the fixture pivot axis.

This interface block has a round pin and a blade pin to locate the airfoil mounting adapter in a highly repeatable manner. This two element method of locating parts on a fixture is very effective when two holes with parallel centerlines are available. One hole is designated the primary location reference and mates with a round pin. The second hole aligns the part. Since there is always a dimensional tolerance in the hole to hole distance, the second pin cannot be round if there is to be a close fit between the hole and the pin. Instead, a diamond or blade shaped pin is used. The blade is placed perpendicular to a line between the hole centers. This configuration permits some variation in the hole to hole distance without any appreciable angular variation. Hardened, diamond shaped pins (relieved bullet nosed pins) are available commercially in fractional sizes for this purpose to mate with hardened bushings. For the odd sizes encountered on this test part, the blade configuration was used.

DATUM LOCATION The coordinate system datum for this test part is at the intersection of the centerline of the primary mounting hole in the airfoil mounting adapter and the base of the airfoil mounting adapter. Mounting the airfoil on the interface block locates the coordinate system origin at known X, Y, and Z distances with reference to the machine coordinate system and nominal airfoil CG location. The nominal locations are circled on figure 8.

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The actual location of the test part datum was determined by making a tare measurement with the bare interface block and a special mounting screw. The CG Locator Weight, (fig. 9) was then mounted in the Interface block for part measurement. The CG of the Locator Weight is on the centerline of the weight (reference system Z axis) at a known location (Z=+1.386") relative to the center of the cross pin. The cross pin line of contact on the interface block is at Z= 0.000. The cross pin positions the locator weight accurately along the Z axis.
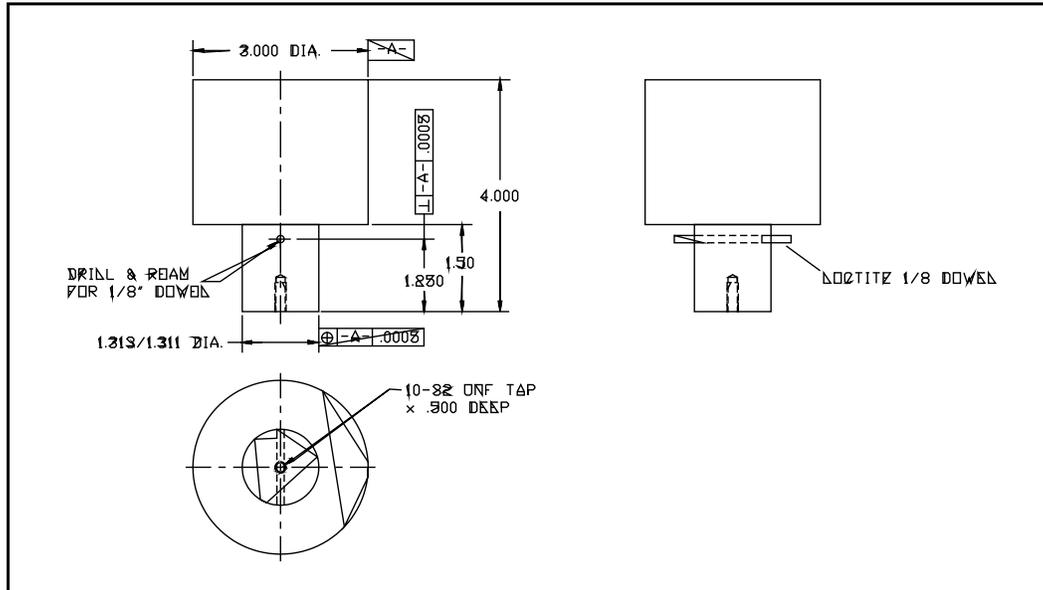
Figure 9 - CG Locator Weight

This known CG location of the weight, relative to the Interface block, along with the measured location of the locator weight CG relative to the machine allowed the coordinate system origin to be referenced to the machine centerline with a high degree of accuracy. The X, Y, and Z origin offsets from the machine centerline were entered into the computer program so the airfoil CG location could be reported with reference to the airfoil coordinate system as well as the machine coordinate system.

The interface block, airfoil mounting bolts, and mating counterweight, were considered as part of the fixture and mounted for each tare measurement. This assembly was attached to the airfoil for all part measurements. Care was taken to insert the airfoil mounting bolts the same number of turns for tare as for part measurements.

   TEST PART ROTATION The interface block is provided with two sets of reamed holes for alignment pins. These alignment pins mate with the three fixture plates, two of which are mounted on pivots, to orient the airfoil in the six test positions. The alignment pins are shoulder screws which allow them to be used both to align and fasten the mating parts.

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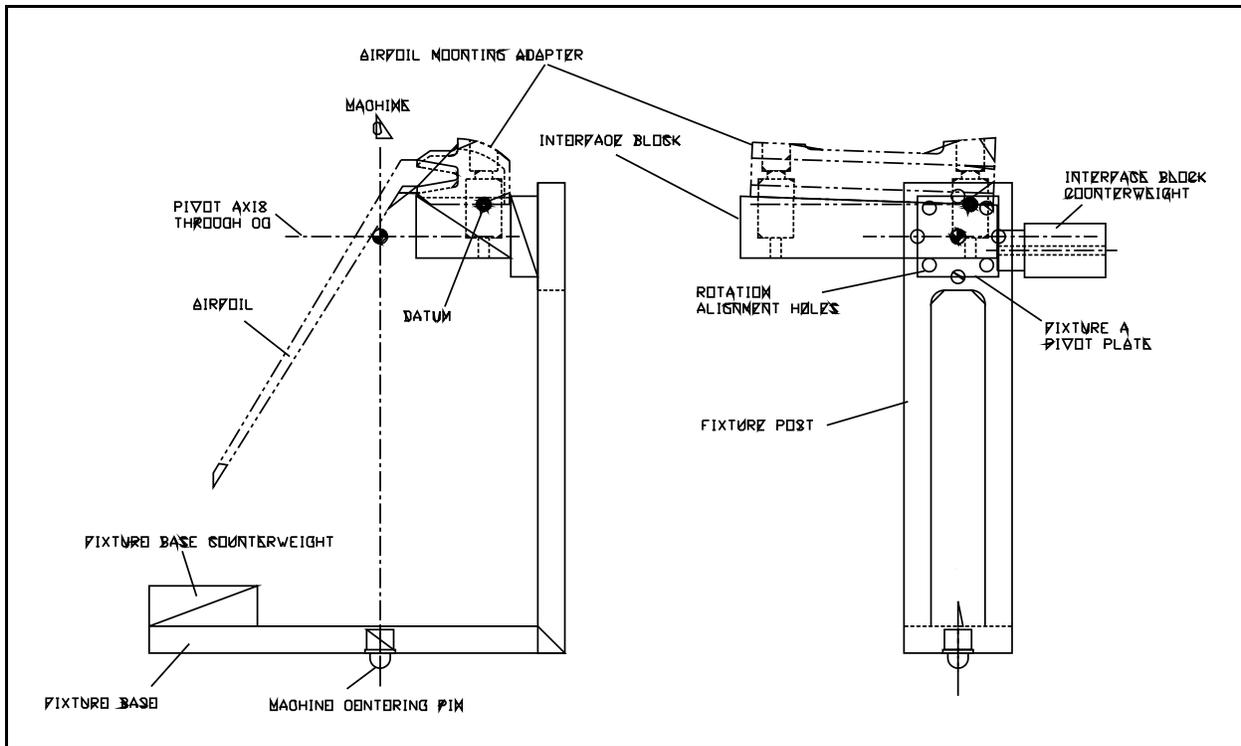
Figure 10 - Fixture -A- Layout

Fixture A (fig. 10) is used to orient the test part for positions 1, 2, and 3. The Fixture A Pivot Plate, to which the interface block is mounted, pivots about the Y axis. Similarly, Fixture B Pivot Plate (fig. 12) is used to rotate the airfoil about the Z axis to positions 4 and 5. Eight reamed holes in the common fixture post permit the fixture plates to be repeatably rotated to 0, 45, and 90 degree orientations and locked in these positions for both tare and part measurement. Note the large lightening cutouts in the post. Other fixture parts are similarly lightened where possible.

The third fixture, figure 11, does not rotate but simply orients the airfoil in position 6. Note that fixture C is much lower than A and B. Since it does not need to provide rotational clearance, the lower height reduces weight and potential dimensional errors.

To provide best accuracy, tare measurements were made for each position prior to, or immediately after, the part measurements. No change in the fixture position occurred between part and tare measurements.

Figure 11

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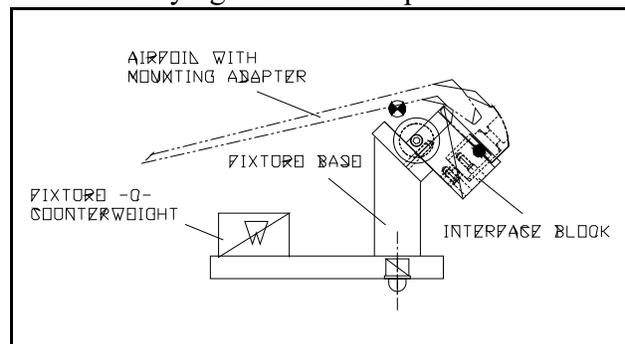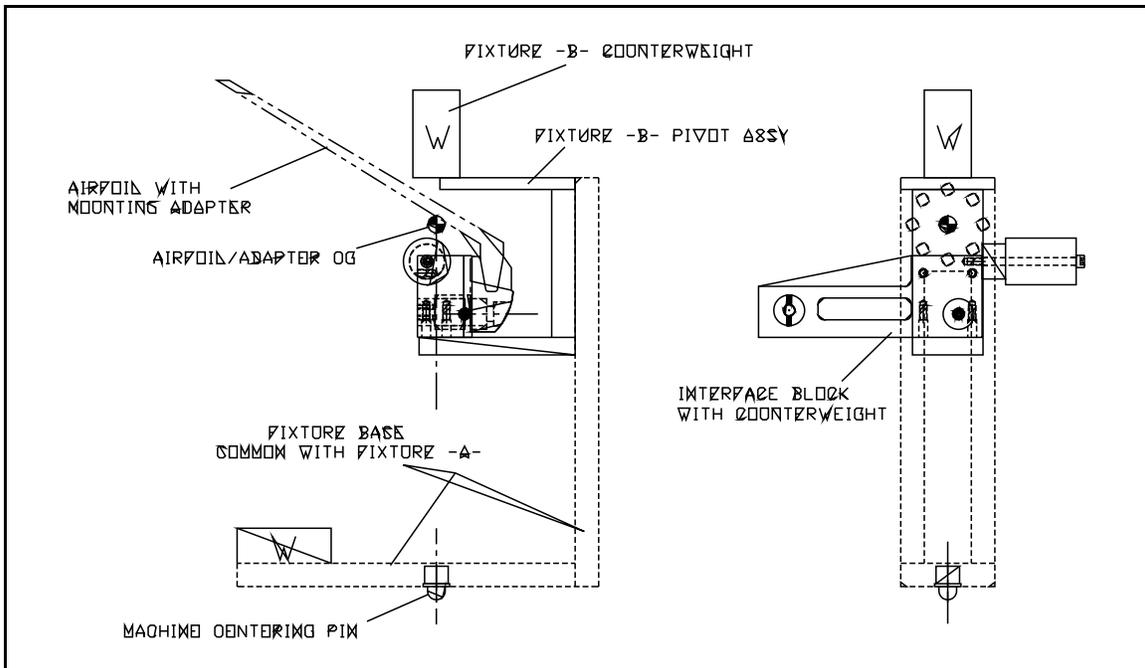
Figure 12 - Fixture -B- Layout

8 HELIUM CHAMBER DESIGN It quickly became obvious that the advantages of the helium method over vacuum measurements were indeed very practical considerations. There are only four requirements for a helium chamber. They are, tight seal, top down flow, operator access, and helium flow control.

A simple inexpensive chamber was constructed of wood and polyethylene film to meet these requirements (figs. 13 & 13A).

1. **Tight seal**. The chamber must be tightly sealed from just below the bottom of the test part to the top of the chamber to insure that the entire test part is enclosed in a helium atmosphere. Helium has a very small molecule and will leak out of any small openings in the upper part of the chamber at a surprisingly high rate. Below the test part, the chamber must exhaust freely to the outside

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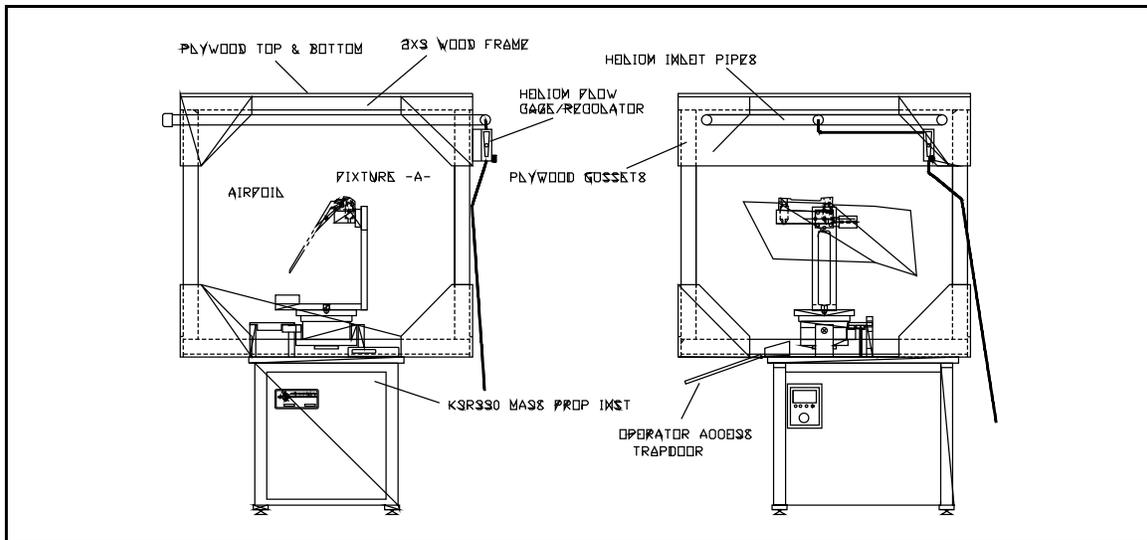
Figure 13

atmosphere to insure that the helium purge is not restricted. This free exhaust should consist of a small (i.e. 1/8 inch) gap around the periphery or a number of holes, not exceeding 1/2 inch diameter in the bottom.

The seal may be accomplished by using (clear plastic) side panels and RTV or self adhesive foam tape. We have used both methods and prefer using a 2 sided 1/8 x 1/2 adhesive foam tape to serve as both fastener and seal. The tape should be applied to the frame in a continuous strip starting at the bottom of each side. It can be contoured or notched at the corners to provide a continuous seal. The frame, of course, must also be sealed. A welded angle iron (or aluminum) frame works very well with the clear panels mounted on the inside. Some grinding may be required to provide a flat surface to mount the panels.

A very inexpensive alternative is to build a simple wooden frame of 2" x 3" framing with a plywood top covered with clear polyethylene film. The film should be one piece, folded to hug the frame and stapled to the bottom frame members. We have used this type of chamber with good results. We did find that a clear window taped into one side was necessary for adequate visibility into the chamber (fig. 13A).

Since there is no significant pressure differential between inside and outside the chamber, chamber strength is dictated only by the requirements imposed by handling the frame. This is in sharp contrast to vacuum chambers and results in major cost savings.

2. **Top down flow.** Since helium has a much lower density than that of air, (about 1/7), the helium should be introduced at the top of the chamber so that the air will be displaced down and out of the chamber with a minimum of turbulence. Helium inlet velocity should be low for the same reason. There is, however, a compromise. If the inflow rate is too low, there will be excessive diffusion and it will be difficult to achieve a high helium concentration. The concentration should be greater than 95% helium for best results. The chambers we have used varied from about 50 to 150 cubic feet. The helium inlet was adjusted in each case to purge the chamber in 12 to 20 minutes. Once the chamber was purged, the flow rate was reduced to a value just sufficient to maintain the concentration.

Figure 13A
Operator and Helium Chamber

The shape of the chamber also influences the purge rate. If the chamber can be built to have a smaller horizontal cross section at the top than at the bottom, there will be less mixing, and the chamber will have less total volume. Both of these factors will reduce the purge time and helium cost, but may increase fabrication cost.

3. **Operator access.** A totally unique attribute of helium testing is that the operator can enter the chamber while the test is in progress to exchange fixtures, or install, adjust or reorient the test part. This turns out to be a major advantage over measuring in a vacuum, reducing both time and cost.

The access should be designed to allow the operator to enter from below, through a trap door. This will minimize turbulence and the introduction of air into the chamber. We found that if the helium purge flow was increased while the operator was in the chamber, very little atmospheric mixing resulted and testing could resume almost immediately.

Helium is not hazardous or poisonous, but neither is it breathable. For this reason, the operator must wear a snorkel connected to the outside atmosphere by means of a flexible hose when entering the chamber. A typical skin diving snorkel with the vertical tube removed can be connected to a piece of flexible corrugated 3/4 PVC tubing. The free end of the tube is fastened outside the chamber (fig. 14).

The exhaust of helium into the room where tests are conducted is not normally a problem. The typical helium source is a nominal 300 cubic foot compressed gas bottle. Oxygen concentrations of less than 16 percent (84% air) are considered potentially asphyxiating and concentrations below 19% potentially dangerous for people with heart or lung problems. In order to reduce the oxygen concentration from the normal 21% to less than 19%, the room air would have to have a helium concentration of greater than 10%. Looking at it another way, the test room would have to be smaller than 3000 cubic feet (20 x 15 x 10 ft high) if the entire 300 cubic foot helium bottle were suddenly released. Most mass properties labs are much larger than this, and sudden release of a full gas bottle would not go unnoticed.

There is also a built-in safety feature since high helium concentrations become very obvious when everyone starts talking like Donald Duck!

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Helium, as normally provided in 2400 psi tanks has almost no water vapor in it. This very dry atmosphere may cause eye irritation. For this reason it is advisable that the operator wear a diving mask or goggles. The mask will also prevent accidental breathing through the nose.

Figure 14
Operator and Mask and Snorkel Inside Helium Chamber

4. **Helium flow control.** Helium flow control may as simple as a needle valve and flow meter in the helium line to assure that the purge rate is adequate. At the other extreme, it may involve oxygen concentration sensors, and automatic flow regulation.

Ideally, one would measure and control the helium concentration at about 95%. This is somewhat impractical because helium is inert so there are no simple, inexpensive transducers which are sensitive to helium concentration.

The next best way is to measure the oxygen concentration. A concentration of 1% oxygen is equivalent to a helium concentration of 95%. The difficulty with this system is that oxygen sensors which are accurate at or below 1% are very expensive.

For most applications, a simple needle valve and flowmeter is adequate. The flow rate is set initially to purge the air from the chamber in 10 to 20 minutes and then maintain a purge rate sufficient to keep the helium concentration at the desired level. For setup, there are several simple and inexpensive methods to determine when the chamber is full of helium. An ordinary tapered tube (variable area) flowmeter may be used. However, if the flowmeter is calibrated for air, the helium flow rate will be 7 times greater than the value indicated.

The specific system selected will be determined by the required POI and MOI accuracy required, the magnitude of the air effect on a specific test part, and the size of the room in which the tests are run.

9 SOFTWARE The procedure described above to obtain the necessary tare and test part measurements in both air and helium requires careful attention to detail. If the sequence is violated,

tests must be rerun or the results will be in error.

When numerous (identical) parts must be measured, the operator can perform much more reliably if the computer is programmed to prompt the operator through the sequence, store the data appropriately, and retrieve it correctly on demand to perform the necessary calculations.

Operator prompts should take the form of:

MOUNT FIXTURE B IN POSITION 4 FOR MOI TARE MEASUREMENT.
BE SURE PART MOUNTING SCREWS ARE INSERTED.

Press F10 to continue ...

When the specified key has been pressed, a message should be displayed telling what operation the machine is performing. For example:

MOI TARE MEASUREMENT IN PROGRESS.
FIXTURE B, POSITION 4

A drawing, such as figure 5, showing the fixtures and test positions should be displayed prominently in and next to the test chamber as a reference for the operator to cross check his fixtures and test positions during setup and when the operation message is displayed.

10 HELIUM MEASUREMENTS USED TO DETERMINE VACUUM VALUES OF MOI AND POI To determine the MOI values in vacuum our proprietary "helium method" was employed. These vacuum MOI values were used to calculate POI.

The basic relationship between air, helium, and vacuum measurements is shown in the graph, figure 15.
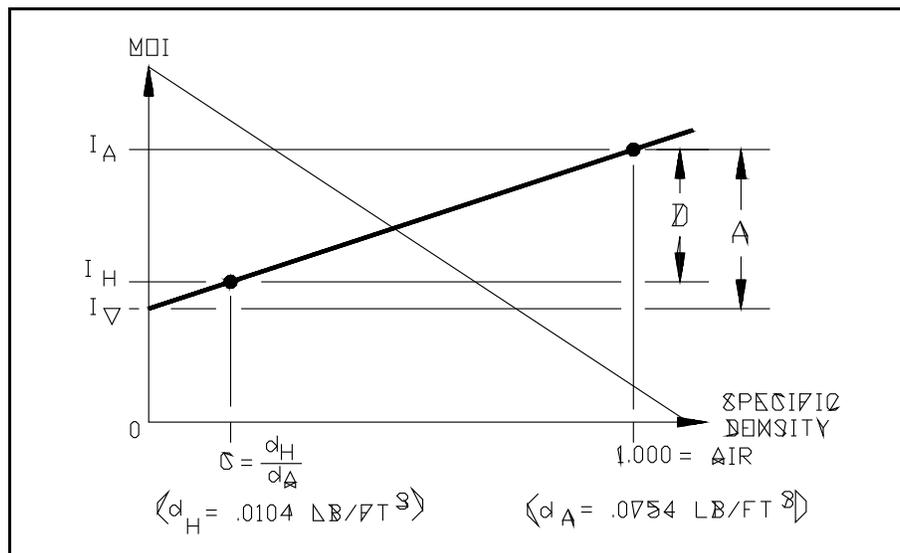
Figure 15

From this graph, the equations relating I_V and I_A can be readily expressed as:

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$$I_V = I_A - A \qquad\qquad (11)$$

and

$$A = \frac{D}{1-C} \qquad\qquad (12)$$

In this equation, I_V is the extrapolated MOI value in vacuum, I_H is the MOI measured in a reduced density atmosphere (helium), I_A is the MOI measured in air, **A** is the air factor or difference between I_A and I_V, **D** is the measured difference between I_A and I_H, and **C** is the specific density, that is, the ratio of the test atmosphere density to normal atmospheric air density. The specific density axis on the graph assigns a specific density value of 1.00 to air.

Combining the two equations expresses I_V as a function of easily measured quantities.

$$I_V = I_A - \frac{D}{1-C} \qquad\qquad (13)$$

To reduce the measurement time and uncertainty in the helium atmosphere measurements, repetitive measurements in both helium and air were made on one airfoil to determine the correction factor (A) for each test position which could then be applied to the MOI measurements in air to determine the vacuum MOI values rather than measure each part in each position in both air and helium. This also allowed more accurate determination of the uncertainty in MOI and POI values.

Measurements were then made in air for all test positions for each airfoil and the appropriate factor applied to determine the vacuum values.

This is more accurate and less time consuming than measuring every test item in both air and helium. Since the "Air Effect Factor" is a function of shape, the variation in this factor between test parts will be negligible.

CALCULATION OF AIR EFFECT FACTOR The composition of the test atmosphere for this test was determined by measuring the oxygen level in the test chamber. The level was maintained for all tests at approximately 0.6% oxygen (average). This equates to 2.89% air and 97.11% helium. At standard conditions, the density of air is .0765 lb/ft^3 and the density of helium is .0106 lb/ft^3. The calculated density of the test atmosphere is therefore;

$$\frac{(2.89 \times .0765) + (97.11 \times .0106)}{100} = .0125 \; lb/ft^3$$

The ratio (C) between the test atmosphere and air is .163. This gives a constant of proportionality (1/(1-C)) = 1.195. This constant is used to multiply the difference (D) between air and helium MOI measurements to determine the air factor.

The effect of minor variations in atmospheric density is a second order effect on the MOI and POI

of the test part. For this reason, it is usually adequate to use a simple flowmeter in the helium purge line to monitor the helium inflow. An initial purge of about one and one half times the chamber volume in about 15 minutes will develop an atmosphere of between 95 and 100 percent helium. The purge rate may then be reduced to introduce no less than one chamber volume per hour to maintain this concentration. Any time the operator enters the chamber to change the test position, the purge rate should be increased to the initial rate and kept at that level while the operator is in the chamber and for one or two minutes after exiting.

Be sure the flowmeter has been calibrated for use in helium. A ordinary tapered tube flowmeter, (variable area meter) if calibrated for air will read only 1/7 the helium volume being delivered. When purchased, the flowmeter used for this exercise was calibrated for air, so a new float was made with the same cross sectional area and length but only 1/7 the weight as the original. The meter was than recalibrated using helium by direct volume displacement.

11 AIRFOIL TEST RESULTS

MOI METHOD FOR POI MEASUREMENT The airfoil in this exercise required determination of all mass properties to within the accuracies stated in paragraph 4. Raw data for air and helium MOI measurements were recorded along with the CG location and weight. The raw air MOI data was then corrected using the air effect factor to determine the vacuum MOI values. The vacuum values were then entered in the computer program and final results of Weight, MOI, CG, and POI were calculated. A sample data correction sheet for one airfoil and the final test results are shown in figures 16 and 17.

AIRFOIL DATA CORRECTION SHEET AIRFOIL S/N 8-A

TEST ATMOSPHERE   98.1 % HELIUM   2.9 % AIR

AIR FACTOR = 1.195 X (MOI AIR - MOI HELIUM)

POSITION	MOI AIR	MOI HELIUM	AIR FACTOR	MOI VACUUM
1 (I_{ZZ})	396.468	395.401	1.275	395.193
2 (I_{45ZX})	339.979	337.794	2.611	337.368
3 (I_{XX})	144.969	142.922	2.446	142.523
4 (I_{YY})	382.242	379.830	2.882	379.360
5 (I_{45})	330.288	328.665	1.940	328.348

Figure 16 - Data Correction Sheet

```
AIRFOIL MASS PROPERTIES
SUMMARY OF RESULTS                    AIRFOIL S/N 8-A

WEIGHT(lb)     10.275

CENTER OF GRAVITY
          CG-X (in)              CG-Y (in)              CG-Z (in)

          -1.564          -3.281          -0.382

MOMENTS OF INERTIA
                      MOI IN AIR          MOI IN VACUUM

      I_ZZ (lb-in²)      396.468              395.193

      I_XX (lb-in²)      144.969              142.523

      I_YY (lb-in²)      382.242              379.360

PRODUCTS OF INERTIA
                      POI IN AIR          POI IN VACUUM

      P_XY (lb-in²)      -66.683              -67.407

      P_YZ (lb-in²)      61.650               63.635
```

Figure 17 - Summary of Results

     SPIN BALANCE MACHINE METHOD To confirm our assumption that test parts with significant entrapped and entrained air effects cannot be accurately measured on a POI spin balance machine, we ran the sample airfoil of figures 16 and 17 on the Space Electronics' Model POI-50Z spin balance machine. The results showed that for an airfoil where large aerodynamic forces exist, POI measurements vary widely on the POI machine and were highly dependent on the conditions of enclosure of the test part.

The airfoil was mounted in test position 3 as shown in figure 5. Fixture B, rather than fixture A was used for this test because it was better suited to be dynamically balanced. Three tests were run, each one three times.

    Test 1   - Airfoil in air without the shroud (chamber) in place
    Test 2   - Airfoil in air with the shroud (chamber) in place
    Test 3   - Airfoil in helium using the same shroud as a helium chamber

The results are summarized in the table of figure 18.

MOI METHOD	P_{XZ}	P_{XY}
MEASURED AIR VALUE	-69.261	-66.683
MEASURED HELIUM VALUE	-68.578	-66.325
EXTRAPOLATED VACUUM VALUE	-68.510	-67.407
SPIN METHOD		
AIR W/O SHROUD	-70.872	-67.864
AIR WITH SHROUD	-68.252	-70.987
SHROUD WITH HELIUM	-67.469	-72.172

**Figure 18 - Comparison Between Spin Balance
And MOI Method Of POI Determination**

The data using the spin method is the average of 3 measurements. Within each group of measurements the scatter was typically 0.1 to 0.3 % with one exception. The data for measurements in air without the shroud varied almost +/- 1% from the average for P_{XY}.

All spin test data was taken at 50 RPM in a clockwise direction.

While this limited data does not prove or disprove any theories, the basic agreement in magnitude does show that the MOI method is valid for this type of test part. The shape of this particular airfoil generates large aerodynamic forces in the plane that affects P_{XY} and much smaller forces in the plane that affects P_{XZ}. It would therefore seem logical that the P_{XY} values would have the most scatter, the least agreement between the two methods, and that P_{XZ} would have the best agreement. This is in fact the case.

12 ERROR ANALYSIS

CG LOCATION uncertainties due to machine and fixture non-repeatability were reduced to less than 0.001" by measuring a new tare for each part measurement. The largest source of uncertainty is in the absolute location of the airfoil datum relative to the machine center. Measurements of the datum location indicate repeatability on the order of 0.001" but uncertainty in absolute location is on the order of 0.002" for X and Y. The uncertainty in absolute location of the Z component is on the order of 0.004". Summing these uncertainties gives a worst case uncertainty of CG location relative to the Airfoil datum of **0.003" for X and Y with 0.005" for Z.** The "As Reported" uncertainty is specified as "within" ±0.01 inch rather than the actual calculated uncertainty for each axis.

The uncertainties in CG location have no effect on the accuracy of the MOI measurements since all MOI measurements used in calculations were through the CG without regard to the absolute location of the CG. Furthermore, the smallest radius of gyration (K=MOI/W) in any of the test positions was approximately 4 inches. For a CG location error of as much as 0.01, the MOI error ($4^2/0.001^2$) would still be only 1 part in 160,000 or 0.0006 percent.

MOI MEASUREMENTS also required a new tare for each part measurement to keep the uncertainty **less than ±0.1%** for any one measurement. This is the basic uncertainty of MOI measurement for the machine when adequate care in setup is taken.

ENTRAPPED/ENTRAINED AIR EFFECT varied slightly for different airfoils in the same position. This variation was on the order of 5% and was due to slight differences in shape, weight, area, test atmosphere density etc. The maximum effect for any airfoil in any position, however, was less than 1.5% of the air measurement. Therefore, the uncertainty in any measurement due to the variation in air effect was less then 0.05 x 0.015 or **less than 0.08%** of the air measurement.

In another similar project, the airfoils were considerably smaller and therefore had smaller air effect factors. There was, of course, greater percentage variation in the air effect factor.

By establishing the air effect factor and its uncertainty for each position and using the same factor for all airfoils, the uncertainty of the calculated MOI in vacuum was reduced to approximately 0.2% and POI to approximately 0.4%.

TEST ATMOSPHERE DENSITY was determined by measuring the oxygen level in the test chamber. The level was maintained for all tests at approximately 0.6% oxygen. This equates to 2.89% air and 97.11% helium with a density of 0.0125 lb/ft^3. The uncertainty in atmospheric density is on the order of 0.001 lb/ft^3. This variation would cause less than 0.02% uncertainty in the calculated vacuum MOI.

CONCLUSIONS Can the MOI method be used for a wide range of parts (eliminating the need for a spin balance machine)? The problem is that the object must be oriented accurately in 6 different positions. For many objects such as long rockets, it would be a formidable task to design and fabricate the fixtures. Furthermore, the cost of the fixturing would make this method more expensive than a spin balance machine for general purpose testing. There is a much greater likelihood of error in this method, since there is a chance of either the tare or fixture position being incorrect, and considerable math is required. Some improvement in accuracy can be achieved by programming the computer in the instrument to instruct the operator step by step, and to make the calculations. There is a very steep learning curve for this method. The first few measurements may take you days. Once you have worked this process through successfully, then future measurements can be accomplished in as little as 3 hours per payload. In contrast, spin balance measurements can be made in less than 1/2 hour, including the time to load the payload and get it centered properly. And finally, the sensitivity of this method is not as good as the spin balance method. This method will never replace spin balancing for turbine shafts, other rotating machinery, and high density aerospace objects. For the bulk of aerospace test parts with low frontal area to weight ratios where the entrapped/entrained air effect and aerodynamic forces are small, the standard spin test methods for POI determination will provide more sensitivity and higher accuracy results.

The MOI Method described in this paper works well for large lightweight objects which have significant windage. Here are some observations:

- POI of airfoils is best measured using the MOI method.

- The accuracy of spin test POI measurements is suspect for airfoils.

- The scatter in spin test measurements is reduced by using a helium atmosphere.

- We intend to acquire further spin test data to confirm the accuracy of airfoil POI using the MOI method.